

# Optimal functional representations using weighted regularization

## Scientific Achievement

We reconstruct data from sparse coefficients  $c \in \mathbb{R}^{N \times l}$  from measurements  $u \in \mathbb{R}^{m \times l}$  and basis  $A \in \mathbb{R}^{m \times N}$  :

$$u \approx Ac \longrightarrow \begin{matrix} \text{[vector]} \\ \text{[matrix]} \\ \text{[vector]} \end{matrix} = \begin{matrix} \text{[matrix]} \\ \text{[matrix]} \\ \text{[vector]} \end{matrix}$$

- Possible to design optimal representations given limited number of measurements:  $m \ll N$ .
- Capable of both sparse and joint sparse optimization of data

## Significance and Impact

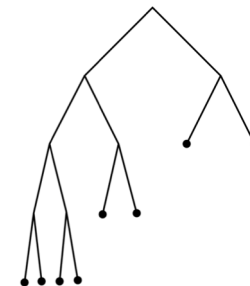
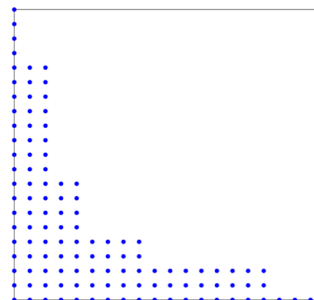
Demonstrate superior compression of data using optimal weighted regularization, exploiting sparsity structures in high dimensional approximation.

## Research Details

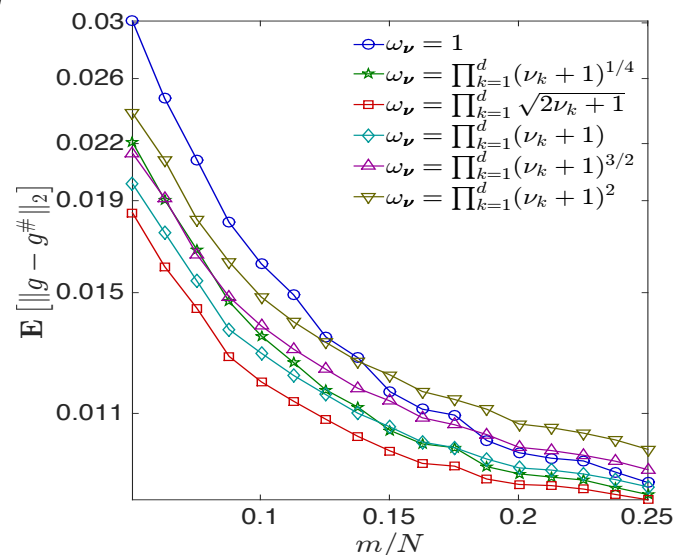
Recovery via regularizations enforcing sparsity:

$$c = \operatorname{argmin} R(z) \quad \text{subject to} \quad u \approx Az$$

$$R(z) = \|z\|_{\omega,1} \quad \text{with} \quad \omega_j = \max |A_{:,j}|$$



Data from UQ and imaging applications often possess downward closed and tree structure.



A comparison of weighted  $l_1$  minimization with different choices of weights

A. Chkifa, N. Dexter, H. Tran, and C. Webster, *Polynomial approximation via compressed sensing of high-dimensional functions on lower sets*. **Math. Comp.** (2017) <https://doi.org/10.1090/mcom/3272>